EES351 2020/1 Part II.3 Dr.Prapun

4.4 Classical DSB-SC Modulators

To produce the modulated signal $A_c \cos(2\pi f_c t) m(t)$, we may use the following methods which generate the modulated signal along with other signals which can be eliminated by a bandpass filter restricting frequency contents to around f_c.

4.55. Multiplier Modulators [6, p 184] or **Product Modulator**[3, p 180]: Here modulation is achieved directly by multiplying $m(t)$ by $\cos(2\pi f_c t)$ using an analog multiplier whose output is proportional to the product of two input signals.

- Such a multiplier may be obtained from
  (a) a variable-gain amplifier in which the gain parameter (such as the the β of a transistor) is controlled by one of the signals, say, $m(t)$. When the signal $\cos(2\pi f_c t)$ is applied at the input of this amplifier, the output is then proportional to $m(t)\cos(2\pi f_c t)$.
  (b) two logarithmic and an antilogarithmic amplifiers with outputs proportional to the log and antilog of their inputs, respectively.
    - Key equation:
$$A \times B = e^{(\ln A + \ln B)}.$$

4.56. When it is easier to build a squarer than a multiplier, we may use a **square modulator** shown in Figure 25.

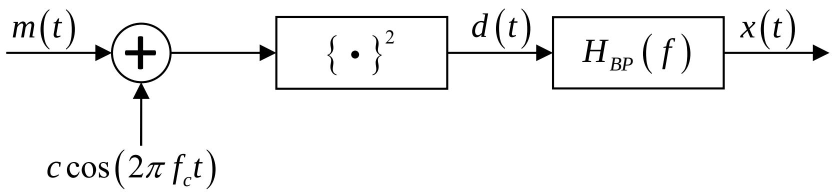
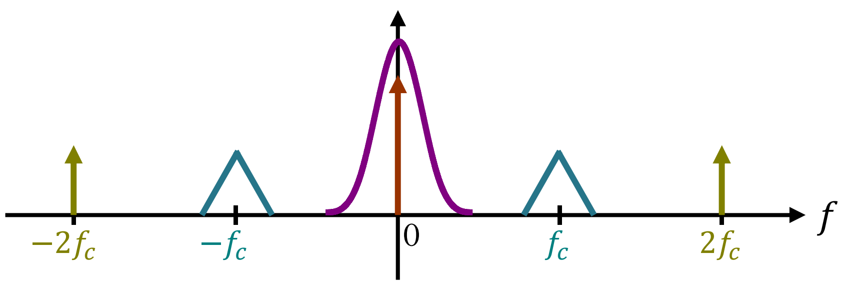


Figure 25: Block diagram of a square modulator

Note that

$$\begin{aligned} d(t) &= (m(t) + c\cos(2\pi f_c t))^2 \\ &= m^2(t) + 2cm(t)\cos(2\pi f_c t) + c^2\cos^2(2\pi f_c t) \\ &= m^2(t) + 2cm(t)\cos(2\pi f_c t) + \frac{c^2}{2} + \frac{c^2}{2}\cos(2\pi(2f_c)t) \end{aligned}$$



Using a band-pass filter (BPF) whose frequency response is

$$H_{BP}(f) = \begin{cases} g, & |f - f_c| \le B, \\ g, & |f - (-f_c)| \le B, \\ 0, & \text{otherwise}, \end{cases} \tag{59}$$

we can produce $2cgm(t)\cos(2\pi f_c t)$ at the output of the BPF. In particular, choosing the gain g to be $(c\sqrt{2})^{-1}$, we get $m(t) \times \sqrt{2}\cos(2\pi f_c t)$.

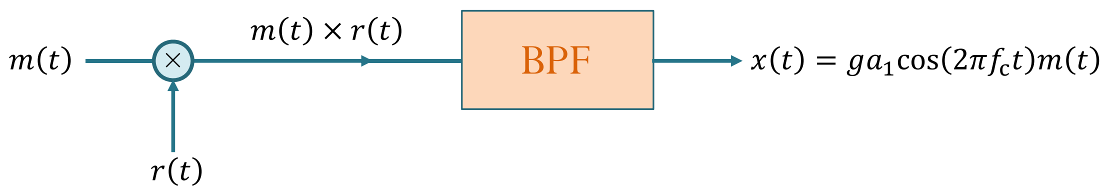
- Alternative, can use $\left(m(t) + c\cos\left(\frac{\omega_c}{2}t\right)\right)^3$.

4.57. Another conceptually nice way to produce a signal of the form $A_c m(t)\cos(2\pi f_c t)$ is to

(1) multiply $m(t)$ by "any" **periodic and even** signal $r(t)$ whose period is $T_c = \frac{1}{f_c}$

and then

(2) pass the result though a BPF used in (59).

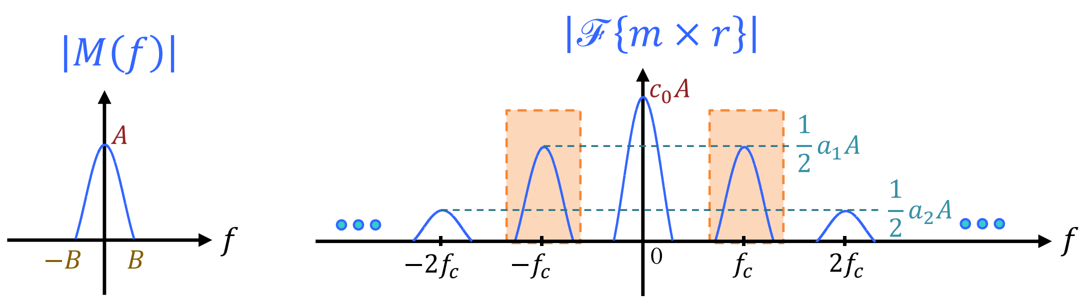


To see how this works, recall that because $r(t)$ is an even function, we know that

$$r\left(t\right) = c_0 + \sum_{k=1}^{\infty} a_k \cos\left(2\pi(kf_c)t\right) \text{ for some } c_0, a_1, a_2, \ldots.$$

Therefore,

$$m(t)r\left(t\right) = c_0 m(t) + \sum_{k=1}^{\infty} a_k m(t)\cos\left(2\pi(kf_c)t\right).$$

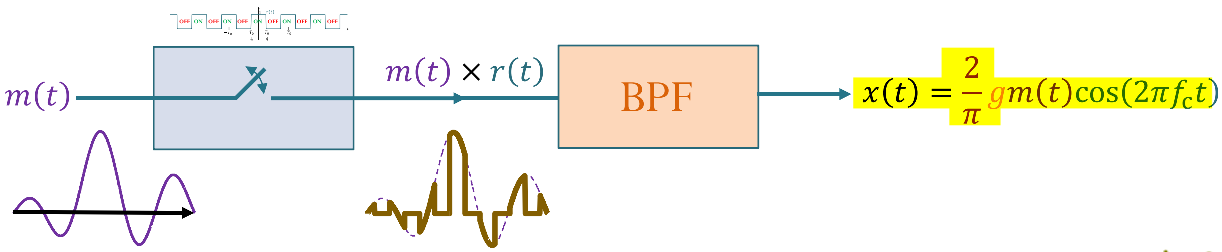


See also [5, p 157]. In general, for this scheme to work, we need

- $a_1 \neq 0$ period of r;
- $f_c > 2B$ (to prevent overlapping).

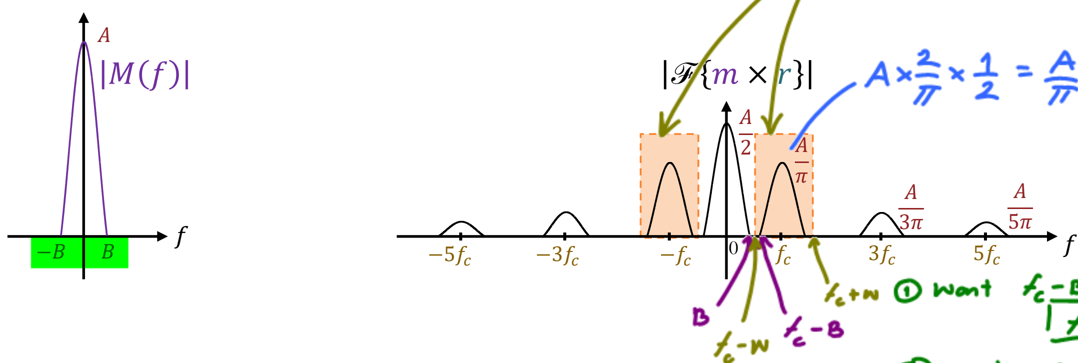
Note that if $r(t)$ is not even, then by (50c), the resulting modulated signal will have the form $x(t) = ga_1 m(t)\cos(2\pi f_c t + \phi_1)$.

4.58. ***Switching modulator***: An important example of a periodic and even function $r(t)$ is the square pulse train considered in Example 4.48. Recall that multiplying this $r(t)$ to a signal $m(t)$ is equivalent to switching $m(t)$ on and off periodically.

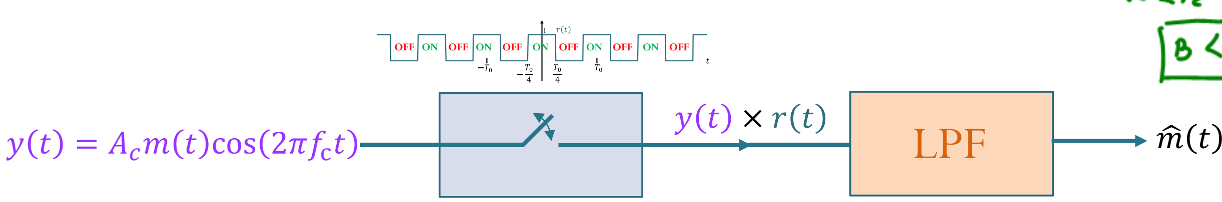


$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$m(t) \times r(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(2\pi f_c t) - \frac{2}{3\pi}m(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}m(t)\cos(2\pi(5f_c)t) + \cdots$$



4.59. ***Switching Demodulator***: The switching technique can also be used at the demodulator as well.



Here, the frequency response of the LPF is

$$H_{\mathrm{LP}}(f) = \begin{cases} g, & |f| \le B, \\ 0, & |f| > B. \end{cases}$$

$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$y(t)r(t) = \frac{1}{2}y(t) + \frac{2}{\pi}y(t)\cos(2\pi f_c t) - \frac{2}{3\pi}y(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}y(t)\cos(2\pi(5f_c)t) + \cdots$$

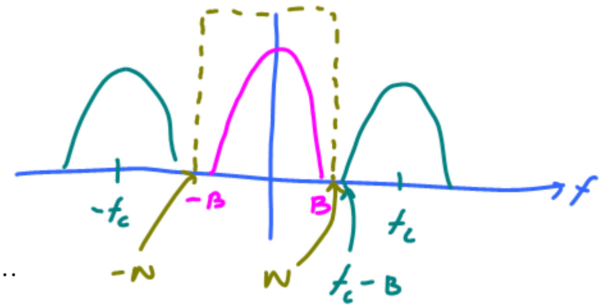
$$= \frac{1}{2}A_c m(t)\cos(2\pi f_c t) + \frac{2}{\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t) - \frac{2}{3\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi(5f_c)t) + \cdots$$

$$\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

$$= \frac{1}{2}A_c m(t)\cos(2\pi f_c t) + \frac{1}{\pi}A_c m(t)\left(1+\cos(2\pi(2f_c)t)\right) - \frac{1}{3\pi}A_c m(t)\left(\cos(2\pi(2f_c)t)+\cos(2\pi(4f_c)t)\right) + \frac{1}{5\pi}A_c m(t)\left(\cos(2\pi(4f_c)t)+\cos(2\pi(6f_c)t)\right) + \cdots$$

② $B < W < f_c - B$

$$= \frac{1}{2}A_c m(t)\cos(2\pi f_c t) + \frac{1}{\pi}A_c m(t) + \frac{1}{\pi}A_c m(t)\cos(2\pi(2f_c)t) - \frac{1}{3\pi}A_c m(t)\cos(2\pi(2f_c)t) - \frac{1}{3\pi}A_c m(t)\cos(2\pi(4f_c)t) + \frac{1}{5\pi}A_c m(t)\cos(2\pi(4f_c)t) + \frac{1}{5\pi}A_c m(t)\cos(2\pi(6f_c)t) + \cdots$$



① Need $f_c - B > B$

$f_c > 2B$

Once again, the switching part is equivalent to multiplying by the $r(t)$ that was used in Example 4.48 and the switching modulator in 4.58. Note that the ON time is synchronized with the nonnegative part of $\cos(2\pi f_c t)$. It can then be shown that if $f_c > 2B$,

$$\hat{m}(t) = \frac{gA_c}{\pi}m(t).$$

We have seen that, for DSB-SC modem, the key equation is given by (41). When switching demodulator is used, the key equation is

$$\text{LPF}\{m(t)\cos(2\pi f_c t) \times 1[\cos(2\pi f_c t) \geq 0]\} = \frac{g}{\pi}m(t) \tag{60}$$

[5, p 162].
Note that this technique still requires the switching to be in sync with the incoming cosine as in the basic DSB-SC.

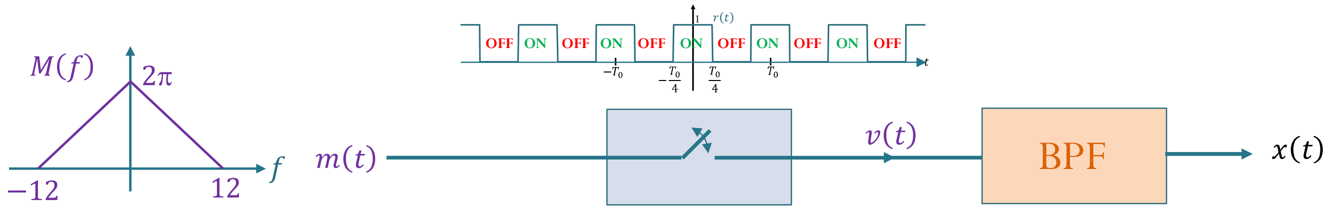
ECS 332: In-Class Exercise # 12 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. [ENRE] Explanation is not required for this exercise.
3. **Do not panic.**

Date: 16 / 10 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. $M(f)$ is plotted on the left below. Consider a switching modulator:

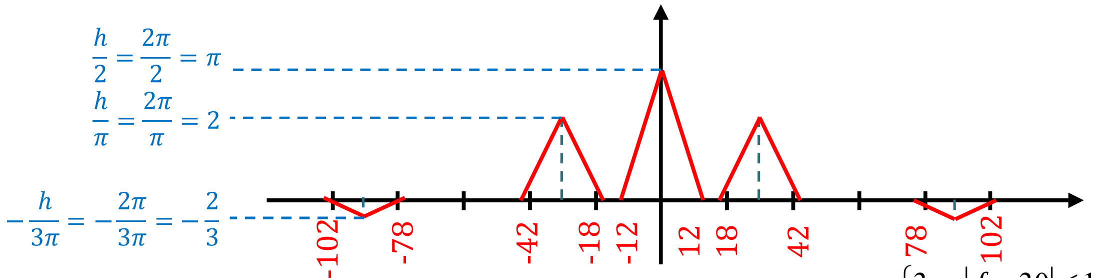


The switching box is operating at frequency 30 Hz with duty cycle 50%.

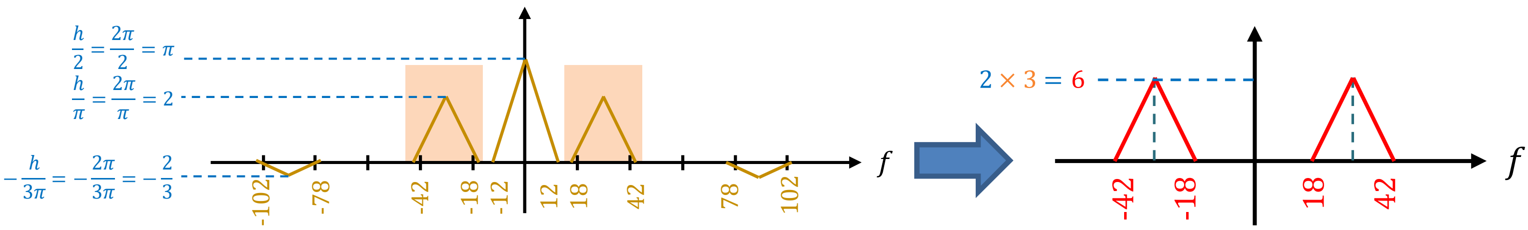
a. Plot $V(f)$

For the BPF, note that $|f-a| \le b$ is the same as $-b \le f-a \le b$ which, in turn, is equivalent to $-b+a \le f \le b+a$.

We have seen, in class, that
$v(t) = m(t) \times r(t)$ where $r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_0 t) - \frac{2}{3\pi}\cos(2\pi(3f_0)t) + \frac{2}{5\pi}\cos(2\pi(5f_0)t) + \cdots$



b. Plot $X(f)$ when the frequency response of the BPF is $H(f) = \begin{cases} 3, & |f-30| \le 14, \\ 3, & |f+30| \le 14, \\ 0, & \text{otherwise.} \end{cases}$ $= \begin{cases} 3, & 16 \le f \le 44, \\ 3, & -44 \le f \le -16, \\ 0, & \text{otherwise.} \end{cases}$



c. Plot $X(f)$ when the frequency response of the BPF is $H(f) = \begin{cases} 4, & |f-33| \le 3, \\ 4, & |f+33| \le 3, \\ 0, & \text{otherwise.} \end{cases}$ $= \begin{cases} 4, & 30 \le f \le 36, \\ 4, & -36 \le f \le -30, \\ 0, & \text{otherwise.} \end{cases}$